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TECHNICAL NOTE

EQUATIONS AND CHARTS FOR

DETERMINING THE HYPERSONIC STABILITY DERIVATIVES OF

COMBINATIONS OF CONE FRUSTUMS COMPUTED BY

NEWTONIAN IMPACT THEORY

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Page 5: In the expression for $C_{m_{\mathbf{q}}}$ (last line of equations (4)) the exponent 2 should appear behind the brackets on the right-hand side. The equation is then correctly written as

$$C_{m_q} = -\frac{8\pi}{sd^2} \sin^2\theta \int_{x_n}^{x_b} x[(x - x_o) + x \tan^2\theta]^2 dx$$

Page 9: In the expression for C_{N_q} (fourth line of equations (9)) the exponent 2 on the quantity r in the first term on the right-hand side should be 3, so that the equation is correctly written as

$$C_{N_q} = \frac{2\pi r^3}{Sd} H^2 \left(1 - \frac{x_{cg}}{r}\right)$$
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NEWTONIAN IMPACT THEORY

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SUMMARY

Equations and charts are presented from which the Newtonian impact theory values of the stability derivatives of cone frustums may be determined for small angles of attack. A procedure is also shown by means of which the coefficients for a missile shape, which is made up of more than one cone frustum or a spherical nose together with one or more cone frustums, can be estimated by impact theory.

INTRODUCTION

The configurations of missiles very often are formed by joining two or more cone frustums end to end. As a first approximation, the stability derivatives at hypersonic velocities for such missile shapes may be estimated from Newtonian impact theory. In view of the simple geometric characteristics of cone frustums, design charts appear to be a convenient method for estimating the Newtonian coefficients for frustums either separately or combined to form a complete missile. The purpose of the present paper is to develop such charts for angles of attack approaching O^O and to give examples of their application for specific missile configurations.

SYMBOLS

smallest diameter of frustum of right circular cone

 C_N normal-force coefficient, $\frac{Normal force}{\frac{1}{2} \rho V^2 S}$

| C_{m} | pitching-moment coefficient, $\frac{\text{Pitching moment}}{\frac{1}{2} \rho V^2 \text{Sd}}$ |
|--|---|
| $^{\mathrm{C}}_{\mathrm{p}}$ | pressure coefficient, $\frac{\text{Pressure differential}}{\frac{1}{2} \rho V^2}$ |
| c_X | axial-force coefficient, $\frac{Axial force}{\frac{1}{2} \rho V^2 S}$ |
| đ | reference diameter; the largest diameter of cone frustum or missile |
| ı | length of frustum or missile |
| q | angular pitching velocity |
| r | radius of spherical nose |
| R(x) | radius of body as function of x (for a cone, $R(x) = x \tan \theta$) |
| | |
| S | reference area, $\frac{\pi d^2}{4}$ |
| s v | reference area, $\frac{\pi d^2}{4}$ free-stream velocity |
| | 4 |
| v | free-stream velocity |
| v v_N | free-stream velocity component of free-stream velocity normal to surface of body |
| v v _N | free-stream velocity component of free-stream velocity normal to surface of body distance rearward from origin of axes |
| $egin{array}{c} egin{array}{c} \egin{array}{c} \egin{array}{c} \egin{array}{c} \egin{array}$ | free-stream velocity component of free-stream velocity normal to surface of body distance rearward from origin of axes distance of moment reference point rearward from nose of body |
| $egin{array}{lll} {\tt V} & & & & & & & & & & & & & & & & & & $ | free-stream velocity component of free-stream velocity normal to surface of body distance rearward from origin of axes distance of moment reference point rearward from nose of body distance of nose of body rearward from origin of axes |
| $egin{array}{lll} {\tt V} & {\tt V}_{ m N} & & & & & & & & & & \\ {\tt x} & & & & & & & & & & & \\ {\tt x}_{\rm cg} & & & & & & & & & & \\ {\tt x}_{\rm n} & & & & & & & & & & \\ {\tt x}_{\rm b} & & & & & & & & & & \end{array}$ | free-stream velocity component of free-stream velocity normal to surface of body distance rearward from origin of axes distance of moment reference point rearward from nose of body distance of nose of body rearward from origin of axes distance of base of body rearward from origin of axes distance of moment reference point rearward from origin of |

λ taper ratio of cone frustum, a/d

ρ mass density of air

angular coordinate in cross-sectional plane of body (see fig. 1)

$$C_{N_{\alpha}} = \left(\frac{\partial C_{N}}{\partial \alpha}\right)_{\alpha \to 0}$$

$$C_{m_{\alpha}} = \left(\frac{\partial C_{m}}{\partial \alpha}\right)_{\alpha \to 0}$$

$$C^{Md} = \left[\frac{9\left(\frac{5\Lambda}{dq}\right)}{9C^{M}}\right]^{d \to 0}$$

$$C_{m_{q}} = \left[\frac{\partial C_{m}}{\partial \left(\frac{qd}{2V}\right)}\right]_{q \to 0}$$

Subscripts:

I, II, III refers to frustum (see fig. 1)

The symbols (s), (f), or (t) following the derivative for a conical frustum denote, respectively, the contribution of the conical surface, of the front face, or the total derivative.

ANALYSIS

Basic Components

Surface of cone frustum. The equations for normal-force, axial-force, and pitching-moment coefficients derived in reference 1 and applied to the conical surface of the frustums of right circular cones for angles of attack smaller than the semiapex angles of the cones are

$$C_{N} = -\frac{2}{S} \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} R(\mathbf{x}) d\mathbf{x} \int_{-\pi/2}^{\pi/2} C_{p} \sin \omega d\omega$$

$$C_{X} = \frac{2}{S} \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} R(\mathbf{x}) \tan \theta d\mathbf{x} \int_{-\pi/2}^{\pi/2} C_{p} d\omega$$

$$C_{m} = \frac{2}{Sd} \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} \left[(\mathbf{x} - \mathbf{x}_{o})R(\mathbf{x}) + \tan \theta R^{2}(\mathbf{x}) \right] d\mathbf{x} \int_{-\pi/2}^{\pi/2} C_{p} \sin \omega d\omega$$
(1)

where \mathbf{x}_n and \mathbf{x}_b are the distances from the origin of axes to the front face and to the base, respectively.

The assumption basic to the Newtonian theory is that when the airstream strikes a solid surface exposed to the flow, it loses the component of momentum normal to the surface and moves along the surface with the tangential component of momentum unchanged. The Newtonian pressure coefficient $C_{\rm D}$ is given by

$$C_p = 2 \frac{V_N^2}{V^2} \tag{2}$$

where the normal component of velocity is given in reference 1 as

$$V_{N} = V \cos \alpha (\sin \theta - \tan \alpha \sin \omega \cos \theta)$$

$$- q \sin \omega \left[(x - x_{O}) \cos \theta + R(x) \sin \theta \right]$$
 (3)

It should be noted that equations (1) are not limited specifically to cones but apply equally as well to any body of revolution which has its total surface area between \mathbf{x}_n and \mathbf{x}_b exposed to the airstream. For a cone, since $R(\mathbf{x}) = \mathbf{x}$ tan θ , the stability derivatives resulting from equations (1) for angles of attack approaching zero become

$$C_{N_{\alpha}} = \frac{4\pi}{S} \sin^{2}\theta \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} \mathbf{x} d\mathbf{x}$$

$$C_{X_{\alpha}} = 0$$

$$C_{m_{\alpha}} = -\frac{4\pi}{Sd} \sin^{2}\theta \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} \mathbf{x} \left[(\mathbf{x} - \mathbf{x}_{o}) + \mathbf{x} \tan^{2}\theta \right] d\mathbf{x}$$

$$C_{N_{\mathbf{q}}} = \frac{8\pi}{Sd} \sin^{2}\theta \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} \mathbf{x} \left[(\mathbf{x} - \mathbf{x}_{o}) + \mathbf{x} \tan^{2}\theta \right] d\mathbf{x}$$

$$C_{N_{\mathbf{q}}} = 0$$

$$C_{M_{\mathbf{q}}} = -\frac{8\pi}{Sd^{2}} \sin^{2}\theta \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} \mathbf{x} \left[(\mathbf{x} - \mathbf{x}_{o}) + \mathbf{x} \tan^{2}\theta \right] d\mathbf{x}$$

$$C_{M_{\mathbf{q}}} = -\frac{8\pi}{Sd^{2}} \sin^{2}\theta \int_{\mathbf{x}_{n}}^{\mathbf{x}_{b}} \mathbf{x} \left[(\mathbf{x} - \mathbf{x}_{o}) + \mathbf{x} \tan^{2}\theta \right] d\mathbf{x}$$

Because the axial-force derivatives due to both angle of attack and pitching are zero by the Newtonian theory, the axial-force coefficient C_X will be discussed hereafter rather than the derivatives. The axial-force coefficient (for $\alpha=0$ and q=0) is given by:

$$C_{X} = \frac{4\pi}{5} \sin^{2}\theta \tan^{2}\theta \int_{x_{n}}^{x_{b}} x dx \qquad (4a)$$

$$R(x)$$

$$X_{n}, x_{0}$$

$$A = \frac{a}{d}$$

Sketch (a)

The stability derivatives l for the sloped sides of a cone frustum, denoted by the symbol (s), are obtained by performing the integrations of equations (4) within the limits shown in sketch (a). It may be noted that the center-of-gravity location is being taken at the front face of the frustum such that $\mathbf{x}_{O} = \mathbf{x}_{n}$. The following equations result:

$$C_{N_{\alpha}}(s) = 2 \cos^{2}\theta (1 - \lambda^{2})$$

$$C_{X}(s) = 2 \sin^{2}\theta (1 - \lambda^{2})$$

$$C_{m_{\alpha}}(s) = -\frac{1}{3 \tan \theta} \left[2(1 - \lambda^{3}) - 3\lambda \cos^{2}\theta (1 - \lambda^{2}) \right]$$

$$C_{N_{q}}(s) = \frac{2}{3 \tan \theta} \left[2(1 - \lambda^{3}) - 3\lambda \cos^{2}\theta (1 - \lambda^{2}) \right]$$

$$C_{m_{q}}(s) = -\frac{1}{6 \sin^{2}\theta} \left[A \cos^{4}\theta + B \cos^{2}\theta + C \right]$$
(5)

where

$$A = 6\lambda^{2}(1 - \lambda^{2})$$

$$B = -8\lambda(1 - \lambda^{3})$$

$$C = 3(1 - \lambda^{4})$$
(5a)

Equations (5) are for the coefficients based on the base area of the cone frustum as the reference area and the base diameter as the reference length. The moment reference point for the C_{m_Q} , C_{N_Q} , and C_{m_Q} equations is the front face of the cone frustum. For a full cone the taper ratio λ is, of course, zero. Of possible use in simplifying computations are the following relationships shown by equations (5):

$$C_{N_{\alpha}}(s) + C_{X}(s) = 2(1 - \lambda^{2})$$

$$C_{N_{q}}(s) = -2C_{m_{\alpha}}(s)$$
(6)

The stability derivatives due to vertical acceleration $\left(\frac{\partial \alpha}{\partial t}\right)$ are shown in reference 1 to be equal to zero by the Newtonian theory; therefore, these derivatives are not considered in the present analysis.

Front face of cone frustum. The flat face of the cone frustum has certain stability derivatives of its own when it is exposed to the airstream. These contributions may be found from equations (5) by allowing λ to go to zero and θ to 90°. When referenced to the base area and base diameter of the cone frustum, the Newtonian values of the stability derivatives due to the front face are:

$$C_{N_{\alpha}}(f) = 0$$

$$C_{X}(f) = 2\lambda^{2}$$

$$C_{m_{\alpha}}(f) = 0$$

$$C_{N_{q}}(f) = 0$$

$$C_{m_{q}}(f) = -\frac{\lambda^{i_{+}}}{2}$$

$$(7)$$

For a flat plate which is perpendicular to the airstream, $C_X = 2$ and $C_{m_q} = -\frac{1}{2}$.

Total derivatives for a cone frustum. - The total derivative may now be written by adding equations (5) and (7):

$$C_{N_{\alpha}}(t) = 2 \cos^{2}\theta(1 - \lambda^{2})$$

$$C_{X}(t) = 2 \left[\sin^{2}\theta(1 - \lambda^{2}) + \lambda^{2}\right]$$

$$C_{m_{\alpha}}(t) = -\frac{1}{3 \tan \theta} \left[2(1 - \lambda^{3}) - 3\lambda \cos^{2}\theta(1 - \lambda^{2})\right]$$

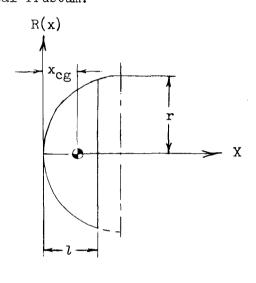
$$C_{N_{q}}(t) = \frac{2}{3 \tan \theta} \left[2(1 - \lambda^{3}) - 3\lambda \cos^{2}\theta(1 - \lambda^{2})\right]$$

$$C_{m_{q}}(t) = -\frac{1}{6 \sin^{2}\theta} \left(A \cos^{4}\theta + B \cos^{2}\theta + C\right) - \frac{\lambda^{4}}{2}$$
(8)

The factors A, B, and C are given by equations (5a).

Cylinders.- A cylinder may be considered to be a special case of a cone frustum for which $\theta=0^{\circ}$ and $\lambda=1.$ Because the upper surface of a cylinder at an angle of attack is not exposed to the airstream, the integrations of the pressure coefficients in equations (1) should be made within the limits $0 \ge \omega \ge -\frac{\pi}{2}.$ Reference 2 contains a comprehensive discussion of the normal force on a cylinder at supersonic speeds and shows that the normal force derived from impact theory is proportional to $\sin^2\!\alpha$ and that the axial force is zero. The integrations of equations (1) when applied to a cylinder give the result that the derivatives $C_{N_{\alpha}}(s), \ C_{X}(s), \ C_{m_{\alpha}}(s), \ C_{N_{q}}(s), \ and \ C_{m_{q}}(s)$ for a cylinder are all equal to zero.

Spherical nose shapes. - In many cases the shape of a missile nose may be approximated more closely by a portion of a sphere than by a conical frustum.



Sketch (b)

The Newtonian values of the stability derivatives for portions of a sphere were obtained from equations (1) by substitution of the relationships

$$R(x) = [x(2r - x)]^{1/2}$$

$$\sin \theta = 1 - (x/r)$$

$$\cos \theta = [(x/r)(2 - x/r)]^{1/2}$$

and by performing the integrations from x_n = 0 to x_b = 1. In this derivation, x_0 = x_{cg} , r is the radius of curvature of the spherical portion, and θ = $tan^{-1} \frac{dR}{dx}$ as shown

in sketch (b). The equations which result are:

$$C_{N_{\alpha}} = \frac{\pi r^{2}}{S} H^{2}$$

$$C_{X} = \frac{\pi r^{2}}{S} H \left[\left(\frac{1}{r} \right)^{2} - 2 \left(\frac{1}{r} \right) + 2 \right]$$

$$C_{m_{\alpha}} = -\frac{\pi r^{3}}{Sd} H^{2} \left(1 - \frac{\mathbf{x}_{cg}}{r} \right)$$

$$C_{N_{q}} = \frac{2\pi r^{2}}{Sd} H^{2} \left(1 - \frac{\mathbf{x}_{cg}}{r} \right)$$

$$C_{m_{q}} = -\frac{2\pi r^{4}}{Sd^{2}} H^{2} \left(1 - \frac{\mathbf{x}_{cg}}{r} \right)^{2}$$

$$C_{m_{q}} = -\frac{2\pi r^{4}}{Sd^{2}} H^{2} \left(1 - \frac{\mathbf{x}_{cg}}{r} \right)^{2}$$

where

$$H = \left(\frac{l}{r}\right)\left(2 - \frac{l}{r}\right) \tag{9a}$$

To base these derivatives on the maximum cross-sectional area and maximum diameter of the spherical segment, use

$$S = \pi r^2 H$$

$$d = 2rH^{1/2}$$

whereupon the following equations result when x_{cg} is taken as zero:

$$C_{N_{CL}} = H$$

$$C_{X} = \left(\frac{l}{r}\right)^{2} - 2\left(\frac{l}{r}\right) + 2$$

$$C_{m_{\alpha}} = -\frac{1}{2} H^{1/2}$$

$$C_{N_{q}} = H^{1/2}$$

$$C_{m_{q}} = -\frac{1}{2}$$

$$C_{m_{q}} = -\frac{1}{2}$$

$$C_{m_{q}} = 0$$

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7

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Of interest is the fact that the damping-in-pitch coefficient $C_{m_{\bf q}}$ of a spherical segment when it is based on its own base area and diameter is constant regardless of the length of the segment. For a hemisphere $\left(\frac{l}{r}=1\right)$, equations (10) reduce to

$$C_{N_{\alpha}} = 1$$

$$C_{X} = 1$$

$$C_{m_{\alpha}} = -\frac{1}{2}$$

$$C_{N_{q}} = 1$$

$$C_{m_{q}} = -\frac{1}{2}$$

$$C_{m_{q}} = -\frac{1}{2}$$

$$C_{m_{q}} = -\frac{1}{2}$$

Equations (10) can then be used in place of equations (8) in the event that the nose of the missile is spherical rather than a cone frustum.

Axis transfer equations. The stability derivatives for a cone frustum may be computed about any point on its axis of symmetry other than on the front face. For example, the transfer equations for the total derivative are given by

$$C_{N_{\mathbf{q}}}(t) = C_{N_{\mathbf{q}_{0}}}(t) - 2 \frac{x_{cg}}{d} C_{N_{\alpha}}(t)$$

$$C_{m_{\alpha}}(t) = C_{m_{\alpha_{0}}}(t) + \frac{x_{cg}}{d} C_{N_{\alpha}}(t)$$

$$C_{m_{\mathbf{q}}}(t) = C_{m_{\mathbf{q}_{0}}}(t) - 2 \frac{x_{cg}}{d} C_{m_{\alpha_{0}}}(t) + \frac{x_{cg}}{d} C_{N_{\mathbf{q}_{0}}}(t) - 2 \left(\frac{x_{cg}}{d}\right)^{2} C_{N_{\alpha}}(t)$$

$$(12)$$

where the zero subscripted derivatives are computed from equations (8) about the front face, x_{cg} is the distance measured rearward positively from the front face, and d is the base diameter. The ratio x_{cg}/d can be written in terms of the length of the cone frustum as

$$\frac{\mathbf{x}_{cg}}{d} = \frac{\mathbf{x}_{cg}}{l} \frac{(1 - \lambda)}{2 \tan \theta}$$

If, for example, it is desired to transfer the moment derivatives to a point one-third of the length rearward from the face, then

$$\frac{\mathbf{x}_{\text{cg}}}{\mathbf{d}} = \frac{(1 - \lambda)}{6 \tan \theta}$$

would be used in equations (12). In the case of a spherical segment,

$$\frac{\mathbf{x}_{\text{cg}}}{\mathbf{d}} = \left(\frac{\mathbf{x}_{\text{cg}}}{l}\right) \left(\frac{l}{r}\right) \mathbf{H}^{-1/2}$$

Combination of Components

In many cases, missile shapes are derived by the joining of two or more cone frustums. In the example used herein, three cone frustums are joined end to end in the manner shown in figure 1. The Newtonian values of the stability derivatives for such a missile shape may be computed from the derivatives of the separate cone frustums in three steps:

- (1) Computation of derivatives for each frustum separately about its own face.
- (2) Transfer of moment derivatives to a common axis for all three frustums.
- (3) Addition of derivatives for separate frustums with proper regard for using the same reference area and diameter.

Computation of derivatives. The stability derivatives of the individual cone frustums are computed separately from equations (8) or (5), depending upon whether the total derivative or only the side contribution is required, or the derivatives may be estimated from the charts presented herein. The derivatives are given subscripts I, II, or III depending upon the frustum to which they pertain.

Transfer of axes.- The moment derivatives calculated in step (1) are transferred to a common axis by means of equations (12). For a missile made up of three cone frustums, the transfer distance x_{cg}/d for each frustum is given in terms of θ and λ by

$$\left(\frac{\mathbf{x}_{cg}}{\mathbf{d}}\right)_{\mathbf{I}} = \frac{1}{2} \frac{\mathbf{x}_{cg}}{l} \left(G_{\mathbf{I}} + \frac{1}{\lambda_{\mathbf{II}}} G_{\mathbf{II}} + \frac{1}{\lambda_{\mathbf{II}}\lambda_{\mathbf{III}}} G_{\mathbf{III}}\right) \\
\left(\frac{\mathbf{x}_{cg}}{\mathbf{d}}\right)_{\mathbf{II}} = \lambda_{\mathbf{III}} \left[\left(\frac{\mathbf{x}_{cg}}{\mathbf{d}}\right)_{\mathbf{I}} - \frac{1}{2} G_{\mathbf{II}}\right] \\
\left(\frac{\mathbf{x}_{cg}}{\mathbf{d}}\right)_{\mathbf{III}} = \lambda_{\mathbf{III}} \left[\left(\frac{\mathbf{x}_{cg}}{\mathbf{d}}\right)_{\mathbf{II}} - \frac{1}{2} G_{\mathbf{II}}\right]$$
(13)

where

$$G_{n} = \frac{1 - \lambda_{n}}{\tan \theta_{n}}$$
 (13a)

and the subscript n refers to the number of the frustum (I, II, or III). If one portion of the missile should be cylindrical, then for that portion

$$G_n = 2 \frac{l_n}{d_n} \tag{13b}$$

where $l_{\rm n}$ is the numerical length of the cylindrical portion. If the nose of the missile should be a spherical segment, then

$$G_{I} = \frac{1 - (1 - H)^{1/2}}{H^{1/2}}$$
 (13c)

where H is given by equation (9a).

Addition of components. The stability derivatives for the individual cone frustums are now referred to a common area and diameter, which in this instance are those for the base of the final frustum of the missile, and added together:

$$C_{N_{\alpha}} = C_{N_{\alpha}}(t)_{1}(\lambda_{II}\lambda_{III})^{2} + C_{N_{\alpha}}(s)_{II}(\lambda_{III})^{2} + C_{N_{\alpha}}(s)_{III}$$

$$C_{X} = C_{X}(t)_{1}(\lambda_{II}\lambda_{III})^{2} + C_{X}(s)_{II}(\lambda_{III})^{2} + C_{X}(s)_{III}$$

$$C_{m_{\alpha}} = C_{m_{\alpha}}(t)_{1}(\lambda_{II}\lambda_{III})^{3} + C_{m_{\alpha}}(s)_{II}(\lambda_{III})^{3} + C_{m_{\alpha}}(s)_{III}$$

$$C_{N_{q}} = C_{N_{q}}(t)_{1}(\lambda_{II}\lambda_{III})^{3} + C_{N_{q}}(s)_{II}(\lambda_{III})^{3} + C_{N_{q}}(s)_{III}$$

$$C_{m_{q}} = C_{m_{q}}(t)_{1}(\lambda_{II}\lambda_{III})^{4} + C_{m_{q}}(s)_{II}(\lambda_{III})^{4} + C_{m_{q}}(s)_{III}$$

In equations (14), it is important to note that the derivatives for the first frustum are calculated from equations (8) and include the contributions of the front face, whereas those for the succeeding frustums are calculated from equations (5) since the forward faces of these portions are shielded from the airflow. Although only in the cases of the C_X and C_{m_q} coefficients do any differences exist between equations (5) and (8), equations (14) are expressed in parallel fashion for consistency.

Charts are presented in figures 2 to 6 giving the variations as functions of θ and λ of the stability derivatives for conical frustums calculated by means of equations (8). These are the total derivatives which include both the front-face and conical-surface contributions. The coefficients are referred to the base area and base diameter of the frustum and to a moment center at its forward face.

It may be noted that the stability derivatives for all cone frustums are completely defined as functions of two parameters, the semiapex angle θ and the taper ratio $\lambda.$ However, the length of the cone frustum (see sketch in fig. 2, for example) will vary with changes in θ and λ according to the equation

$$\tan \theta = \frac{1 - \lambda}{2(1/d)}$$

Thus, for θ = 0 and λ equal to values other than unity, the cone frustum is of infinite length. Caution should be exercised, therefore, in interpreting the significance of the derivatives for extremely small values of θ .

In combining cone frustums into missile shapes, $C_X(t)$ and $C_{m_q}(t)$ for the first frustum must be taken from figures 3 and 6 because the front face is exposed to the airstream. For the subsequent frustums, however, the front faces are shielded from the airstream, and $C_X(s)$ and $C_{m_q}(s)$ must be used. These derivatives, computed from the relationships given in equations (5) are presented as functions of θ and λ in figures 7 and 8.

For illustrative purposes, the effects of shifting the moment reference point rearward one-third the length of the frustum are shown for $C_{m_{\mathbf{Q}}}(t)$, $C_{\mathbf{N}_{\mathbf{Q}}}(t)$, and $C_{m_{\mathbf{Q}}}(t)$ in figures 9, 10, and 11.

The stability derivatives for spherical segments which may be used as missile noses are presented in figure 12. These coefficients were computed by means of equations (10) for the condition $\mathbf{x}_{cg} = 0$ and are based on the base area S and base diameter d of the spherical sector.

A representative missile shape was assumed to be made up of three cone frustums joined end to end in the manner shown in figure 1. Calculations were made to determine the trend of the stability derivatives of the missile as θ and λ were varied separately for each of the three

cone frustums. The results of these calculations are presented in figures 13 to 18. The coefficients are based on the area and diameter of the base of the third frustum and are referred to a moment reference point one-half the length of the entire missile rearward from the nose. It should be noted that as θ or λ was varied for any of the frustums which form the missile shape, the moment reference point was also shifted by virtue of the restriction that $\frac{\mathbf{x}_{\text{CB}}}{7} = \frac{1}{2}$.

If one portion of a missile which is cylindrical should be followed by a conical frustum, then part of the upper surface of the frustum will be shielded from the air flow by the cylinder which precedes it at an angle of attack. Because the results herein are presented for angles of attack which approach zero, however, it is felt that any error associated with the shielding effect will be small. In addition, the effects of centrifugal forces in the flow around bodies of revolution have been neglected. The results presented may be modified to account for the difference between the Newtonian pressure coefficient and the theoretical adiabatic pressure coefficient by the method of reference 3.

CONCLUDING REMARKS

Equations and charts are presented from which the Newtonian impact theory values of the stability derivatives of cone frustums may be determined. The charts are intended to present these derivatives for reasonable values of the length-to-diameter ratio of conical frustums which may be used in missile configurations.

A procedure is also given by means of which the stability derivatives for a missile shape which is made up of more than one cone frustum, or a spherical nose together with one or more cone frustums, can be estimated by impact theory.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., August 12, 1959.

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- 2. Grimminger, G., Williams, E. P., and Young, G. B. W.: Lift on Inclined Bodies of Revolution in Hypersonic Flow. Jour. Aero. Sci., vol. 17, No. 11, Nov. 1950, pp. 675-690.
- 3. Penland, Jim A.: Aerodynamic Characteristics of a Circular Cylinder at Mach Number 6.86 and Angles of Attack Up to 90°. NACA TN 3861, 1957. (Supersedes NACA RM L54A14.)

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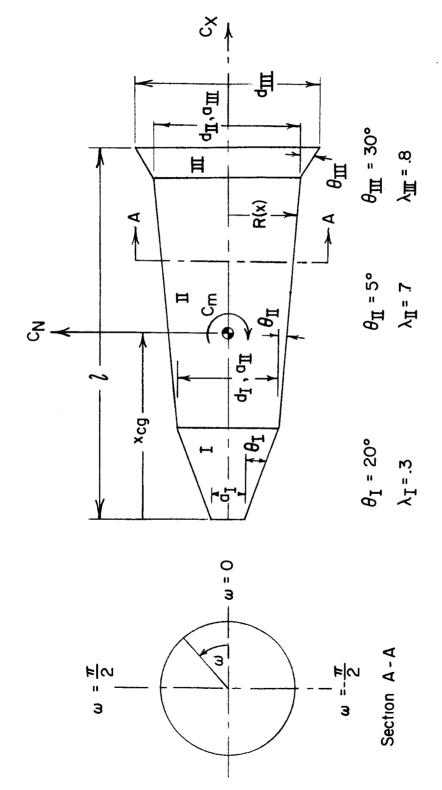


Figure 1. - Representative missile configuration formed from cone frustums.

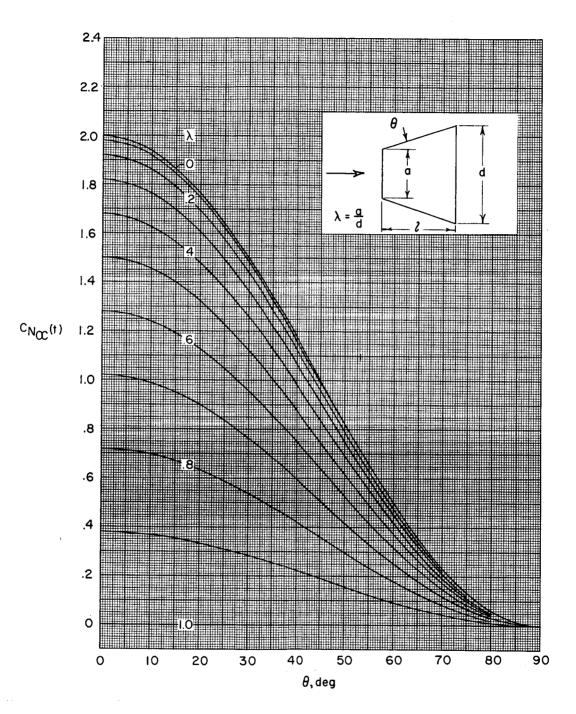


Figure 2.- Total normal-force slope for a cone frustum calculated from Newtonian theory. The geometric parameters are related by the equation tan $\theta = \frac{1-\lambda}{2(1/d)}$.

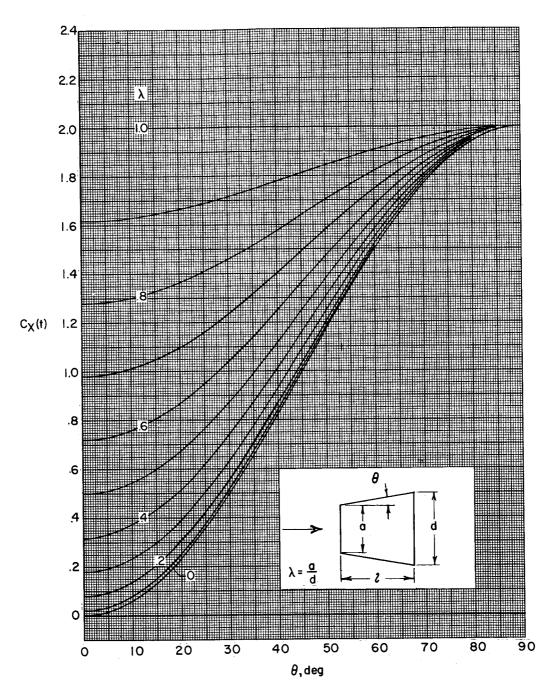


Figure 3.- Total axial-force coefficient for a cone frustum calculated from Newtonian theory. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(1/d)}$.

Figure 4.- Total static-stability coefficient for a cone frustum calculated from Newtonian theory. $\frac{x_{\rm cg}}{l}=0$. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(l/d)}$.

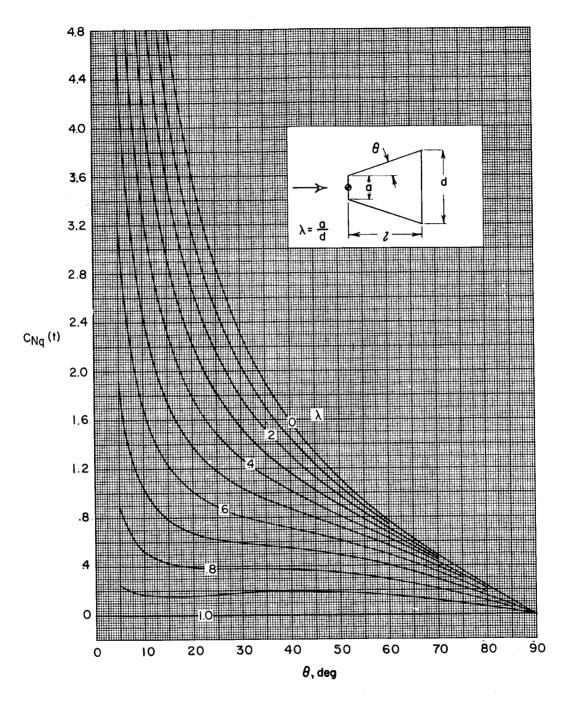


Figure 5.- Total normal-force-due-to-pitching coefficient calculated from Newtonian theory. $\frac{\mathbf{x}_{cg}}{l} = 0$. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(1/d)}$.

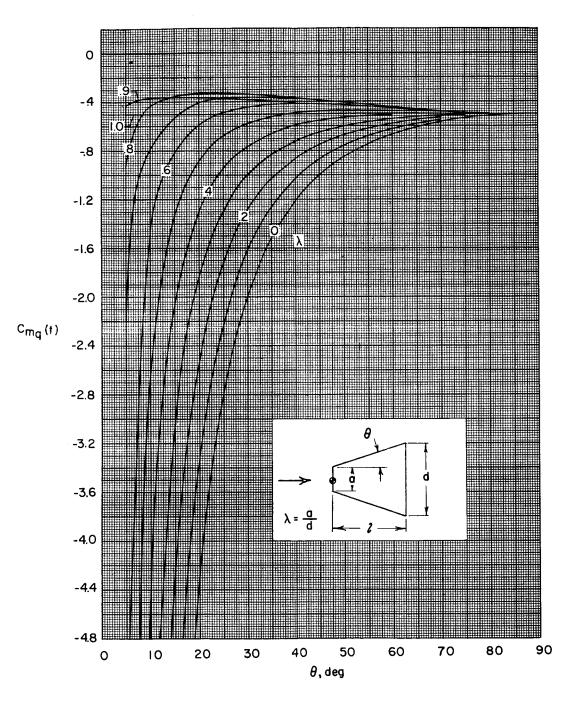


Figure 6.- Total damping-in-pitch coefficient calculated from Newtonian theory. $\frac{x_{cg}}{l} = 0$. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(l/d)}$.

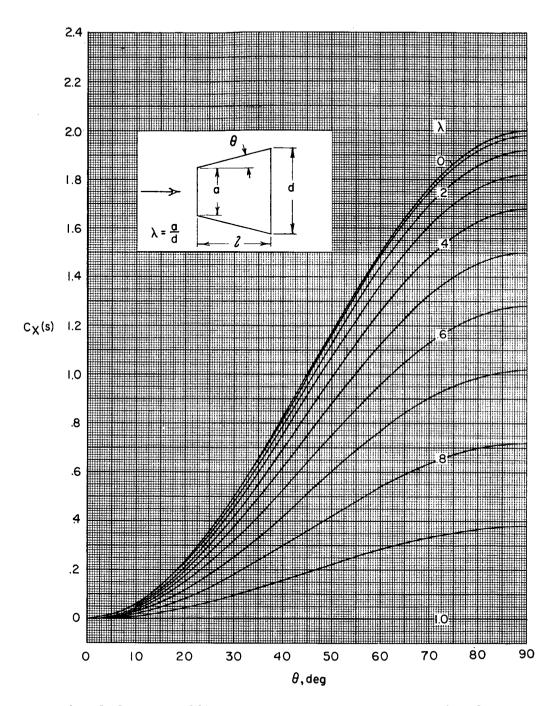


Figure 7.- Axial-force coefficient due only to the inclined sides of a cone frustum calculated by Newtonian theory. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(1/d)}$.

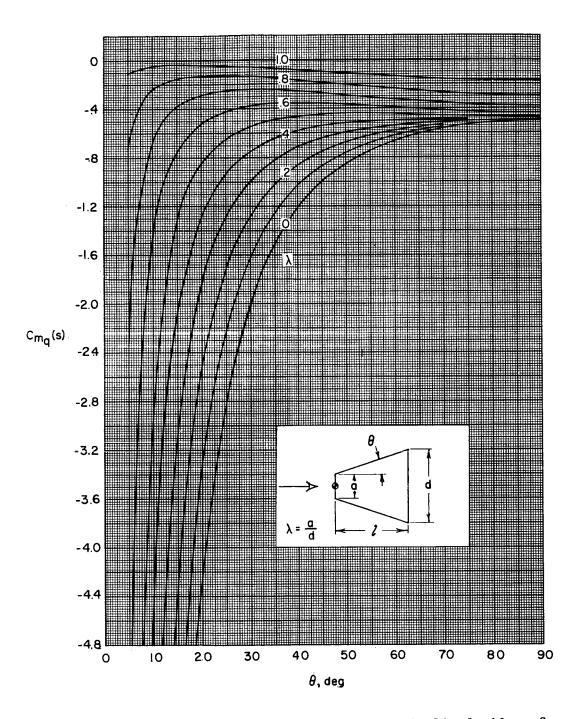


Figure 8.- Damping-in-pitch coefficient due only to inclined sides of a cone frustum calculated by Newtonian theory. $\frac{x_{cg}}{l} = 0.$ The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(1/d)}.$

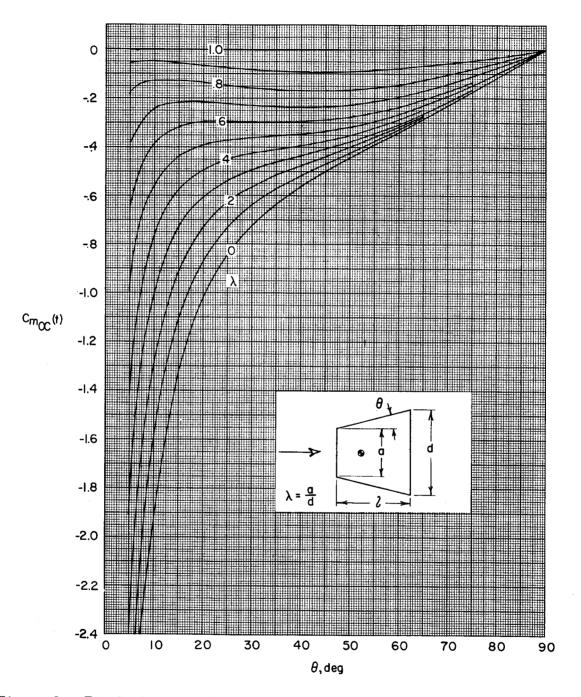


Figure 9.- Total static-stability coefficient for a cone frustum calculated from Newtonian theory. $\frac{x_{Cg}}{l} = \frac{1}{3}.$ The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(1/d)}$.

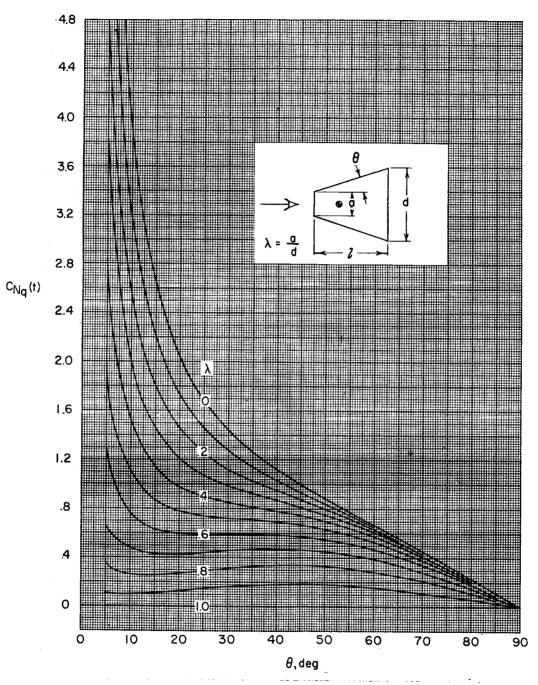


Figure 10.- Total normal-force-due-to-pitching coefficient calculated from Newtonian theory. $\frac{x_{\text{cg}}}{l} = \frac{1}{3}$. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(1/d)}$.

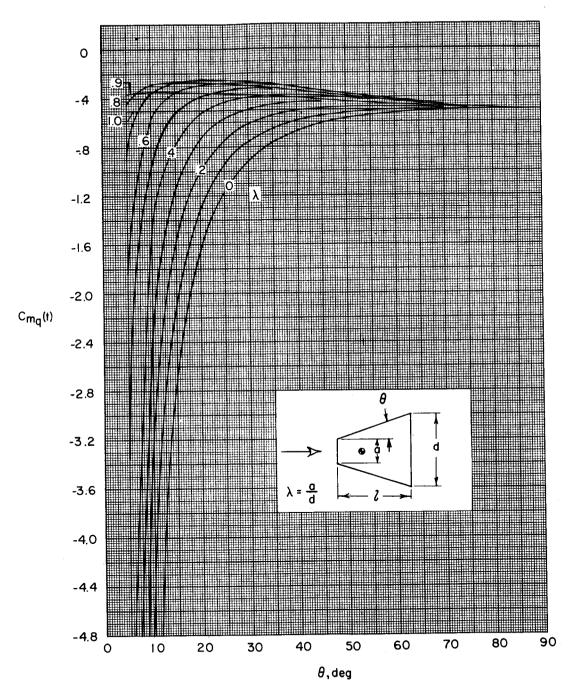
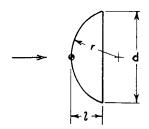


Figure 11.- Total damping-in-pitch coefficient calculated from Newtonian theory. $\frac{x_{cg}}{l} = \frac{1}{3}$. The geometric parameters are related by the equation $\tan \theta = \frac{1-\lambda}{2(l/d)}$.



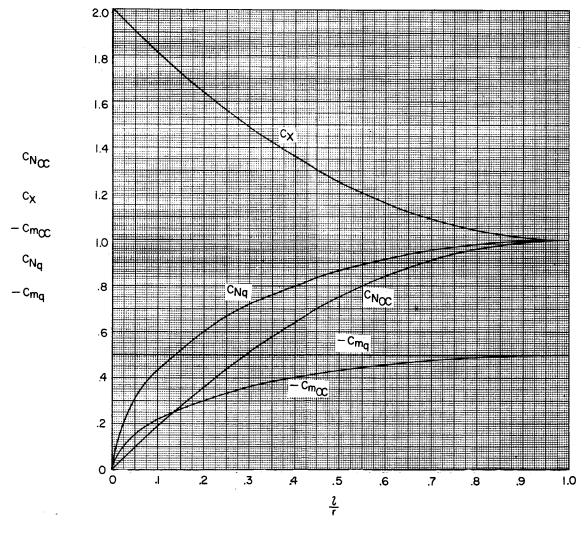


Figure 12.- The Newtonian coefficients for spherical segments referenced to the base area and base diameter of the segment. $\frac{\mathbf{x}_{cg}}{l} = 0$.

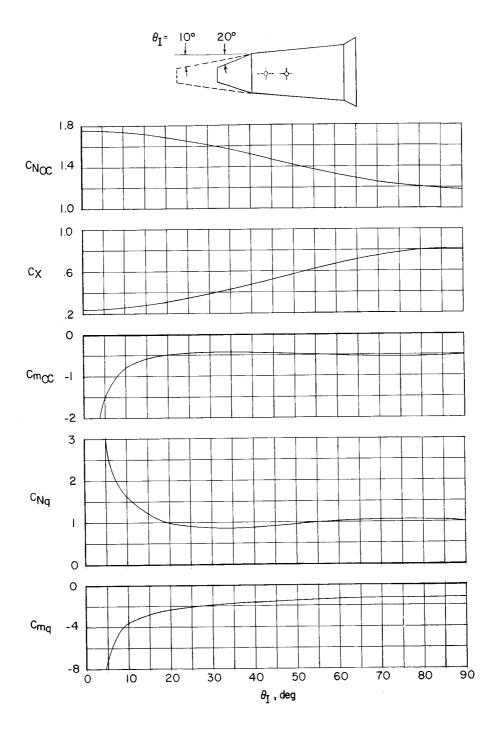


Figure 13.- Effect on Newtonian coefficients of varying $\theta_{\rm I}$ for a missile configuration made up of cone frustums. $\frac{{\bf x}_{\rm cg}}{l}=\frac{1}{2}$.

Figure 14.- Effect on Newtonian coefficients of varying θ_{II} for a missile configuration made up of cone frustums. $\frac{x_{CB}}{l} = \frac{1}{2}$.

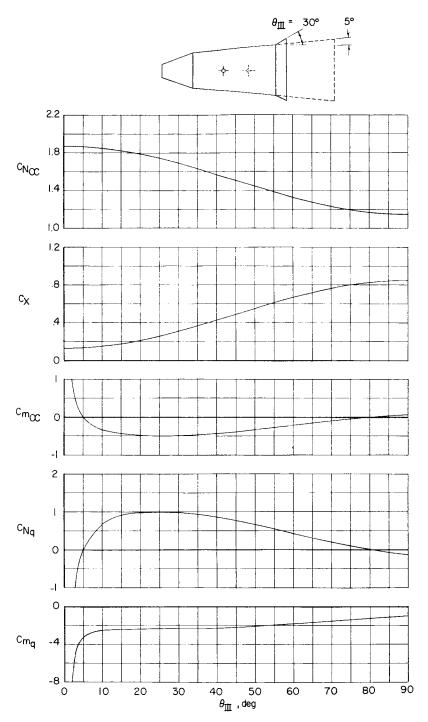


Figure 15.- Effect on Newtonian coefficients of varying θ_{III} for a missile configuration made up of cone frustums. $\frac{\mathbf{x}_{cg}}{l} = \frac{1}{2}$.

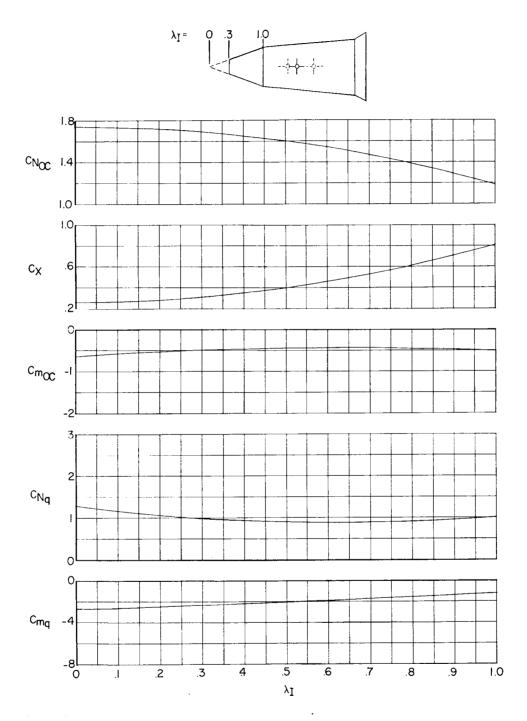


Figure 16.- Effect on Newtonian coefficients of varying $\lambda_{\rm I}$ for a missile configuration made up of cone frustums. $\frac{{\bf x}_{\rm Cg}}{l}=\frac{1}{2}$.

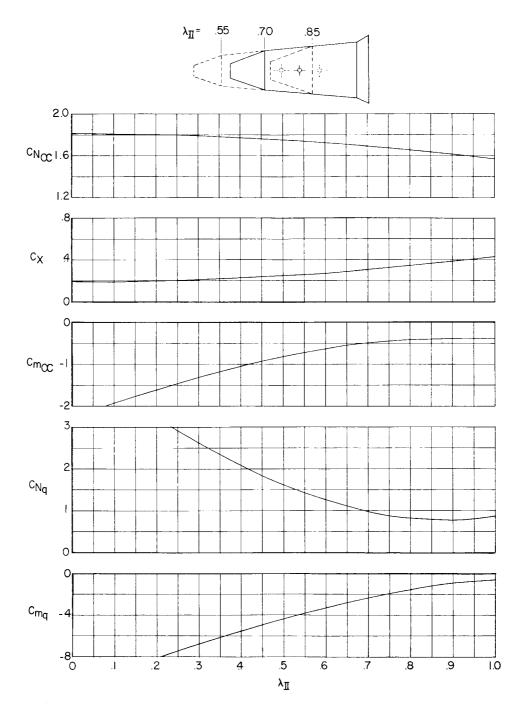


Figure 17.- Effect on Newtonian coefficients of varying λ_{II} for a missile configuration made up of cone frustums. $\frac{\mathbf{x}_{Cg}}{l} = \frac{1}{2}$.

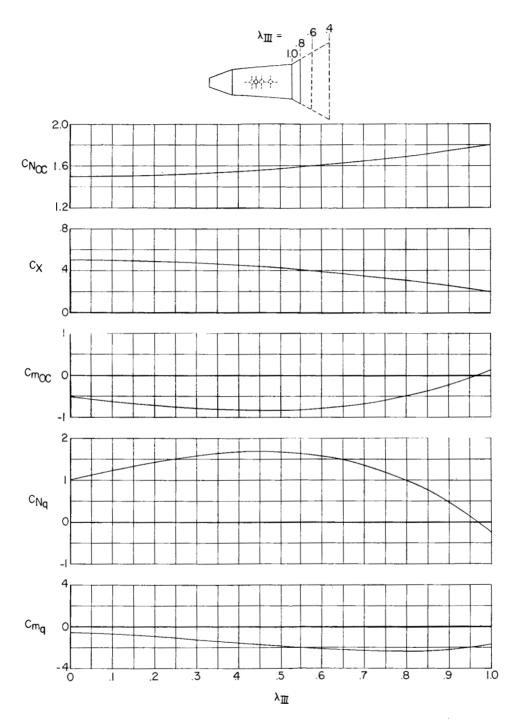


Figure 18.- Effect of Newtonian coefficients of varying λ_{III} for a missile configuration made up of cone frustums. $\frac{\mathbf{x}_{Cg}}{l} = \frac{1}{2}$.